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A GENERALIZED THEORY OF TAPERED TRANSMISSION LINE MATCHING TRANSFORMERS AND ASYMMETRIC 180° COUPLERS SUPPORTING NON-TEM MODES

Protap Pramanick* and Prakash Bhartia**

* COM DEV Ltd.
Cambridge, Ontario

** Dept. of National Defence
Ottawa, Ontario

ABSTRACT

This paper presents a generalized Fourier transform pair for the analysis and synthesis of tapered transmission lines supporting a non-TEM mode. The Fourier transform pair first pointed out by Bolinder, and subsequently used by Klopfenstein for TEM lines can be shown to be a special case of the present transform pair. A step by step synthesis method has been described for exact designs of matching transformers and asymmetric 180° directional couplers.

INTRODUCTION

Tapered transmission lines are useful in the design of matching networks, filters, circulators, etc. The theory of a tapered transmission line supporting a purely TEM-mode is well established [1-3]. Analysis of tapered transmission line supporting TEM modes deals with a constant propagation constant along the taper. Since the propagation constant varies along the length of a non-TEM transmission line taper, the analysis of such tapers becomes involved. So far no rigorous technique has been available for analyzing or synthesizing such tapers.

Microstrip supports a quasi TEM mode at lower microwave frequency bands and a complete hybrid mode at upper frequency bands. Despite the quasi TEM nature of the supported mode the propagation constant of a tapered microstripline varies along its length. Tapered microstrip has been analyzed by Pramanick and Bhartia [4] by solving a pair of complex non-linear differential equations. A similar technique has been applied to unilateral finline, for exponential, cosine squared and parabolic tapers, which support purely non-TEM modes [5]. Schieblich et al [6] reported a synthesis method for optimum Chebyshev unilateral finline tapers. This method carefully avoids the controversial definition problem of the characteristic impedance of a non-TEM transmission line, by synthesizing the taper in terms of a mode coupling coefficient. Moreover, the method is based on certain simplifying assumptions. So far there has been no rigorous theory for designing an optimum Chebyshev dispersive tapered transmission line. This paper presents a generalized Fourier transform pair which is not only mathematically rigorous but also analytically simple to implement even on a desktop computer. The method has been used to obtain the taper profile of a non-TEM, Chebyshev impedance transformer.

THEORY

Consider the schematic representation of a tapered transmission line network in Figure 1. The frequency response F(u) and the taper profile g(p) for such a line can be represented by the following Fourier transform pair [1].

$$F(u) = \int_{-\pi}^{\pi} e^{-jpu} g(p)dp \qquad (1)$$

$$g(p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{jpu} F(u)du \qquad (2)$$

where

$$p = 2\pi\left\{\frac{z}{L} - \frac{1}{2}\right\} \qquad (3)$$

and

$$u = \frac{\beta L}{\pi} \qquad (4)$$

where L is the length of the taper and β is the phase constant. The line is assumed to support a purely TEM-mode where the propagation constant β remains constant along the line.

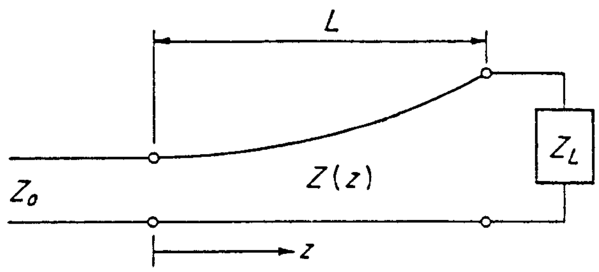

FIGURE 1 : Tapered Transmission Line

The above set of equations can be generalized for non-TEM lines in the following way:

We define:

$$F(\bar{u}) = \int_{-\pi}^{\pi} e^{-j\bar{p}\bar{u}} g(\bar{p}) d\bar{p} \tag{5}$$

and

$$g(\bar{p}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\bar{p}\bar{u}} F(\bar{u}) d\bar{u} \tag{6}$$

where

$$\bar{p} = 2\pi \left\{ \frac{\int_o^z \beta_o(\tau) d\tau}{\int_o^L \beta_o(z) dz} - \frac{1}{2} \right\} \tag{7}$$

$$\bar{u} = \frac{1}{\pi} \int_o^L \beta(z) dz \tag{8}$$

where $\beta_o(z)$ is the propagation constant at a specified design frequency f_o

Now we are in a position to obtain the taper profile $g(\bar{p})$, given the frequency response $F(\bar{u})$, using the Fourier transform pair (5) and (6). Such a method is valid for any frequency response for which the inverse Fourier transform gives a physically realizable taper profile.

On the other hand given a physically realizable taper profile one can always find the frequency response using the modified Fourier transform pair (5) and (6).

In practice most cases require synthesis of tapers rather than their analysis. It is well known that a Chebyshev taper offers the optimum electrical performance. The frequency response for such a taper is given by:

$$\Gamma_i(u) = \frac{1}{2} e^{-j\pi u} F(u) \tag{9}$$

for a TEM line F(u) is given by [7] (See Figure 2).

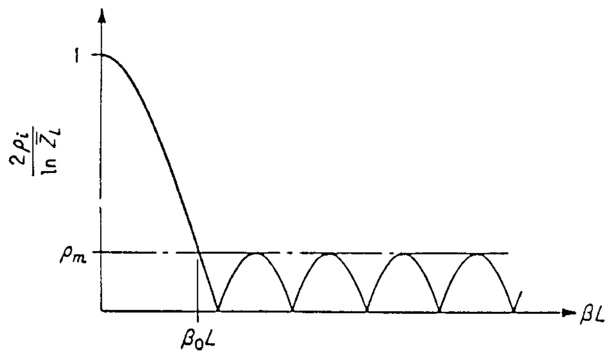

FIGURE 2 : Reflection-Coefficient Characteristic for a Chebyshev Taper

$$F(u) = \ln(\bar{Z}_L) \frac{\cos L\sqrt{(\beta^2 - \beta_o^2)}}{\cosh(\beta_o L)} \tag{10}$$

$$\rho_m = \frac{1}{2} \ln(\bar{Z}_L)/\cosh(\beta_o L), \ \rho_i = \left| \Gamma_i(u) \right|, \ \bar{Z}_L = Z_L / Z_o$$

where u is defined in (4).

For a dispersive line equation (9) assumes the form:

$$\Gamma_i(\bar{u}) = \frac{1}{2} e^{-j\pi\bar{u}} F(\bar{u}) \tag{11}$$

and

$$F(\bar{u}) = \frac{1}{2} \left(Z_L / Z_o \right) \frac{\cos \pi\sqrt{(\bar{u}^2 - \bar{u}_o^2)}}{\cosh(\pi\bar{u}_o)} \tag{12}$$

Substitution of (12) into (6) and the subsequent integration yields the taper profile for Chebyshev response as:

$$\ln(Z(\Theta)/Z_o) = \frac{1}{2}\ln(Z_L/Z_o) + \frac{(\pi\bar{u}_o)^2 \ln(Z_L/Z_o)}{2\cosh(\pi\bar{u}_o)} \Phi\left(\frac{2\Theta}{\pi\bar{u}_o}, \pi\bar{u}_o \right) \tag{13}$$

The function $\Phi(2\Theta/\pi\bar{u}_o, \pi\bar{u}_o)$ is defined to be:

$$\Phi\left(\frac{\Theta}{\pi u_o}, \pi\bar{u}_o\right) = \frac{1}{\pi u_o}\int_o^{\Theta} \frac{I_1\left\{\sqrt{\left\{1-\left(\frac{X}{\pi u_o}\right)^2\right\}}\pi\bar{u}_o\right\}}{\pi u_o\sqrt{\left\{1-\left(\frac{X}{\pi u_o}\right)^2\right\}}}\,dx \quad (13a)$$

where I_1 is the Bessel function of second kind and first order and

$$\Theta = \int_o^z \beta_o(\tau)d\tau - \pi\bar{u}_o/2 \quad (13b)$$

$$-\frac{\pi\bar{u}_o}{2} < \Theta < \frac{\pi\bar{u}_o}{2}$$

and

$$Z(\pi\bar{u}_o/2)/Z_o = exp\left[\frac{\ell n(Z_L/Z_o)}{2cosh(\pi\bar{u}_o)}\right] \quad (14)$$

$$Z(-\pi\bar{u}_o/2)/Z_o = exp\left[ln(Z_L/Z_o) - \frac{ln(Z_L/Z_o)}{2cosh\pi\bar{u}_o}\right] \quad (15)$$

DESIGN STEPS FOR MATCHING TRANSFORMER

1. Given Z_L and Z_o, determine Z(-L/2) and Z(L/2) using (14) and (15)
2. Use a suitable synthesis scheme to obtain the corresponding physical parameter of the transmission line (for example fingap in finline or stripwidth in microstrip).
3. Determine the electrical length $\pi\bar{u}_o$ of the taper from the given ripple level RL in dB.

$$\pi\bar{u}_o = \ell n(R^2 + \sqrt{R^2-1}) \quad (16)$$

$$R = \frac{1}{2}\ell n(Z_L/Z_o)/\rho \quad (17)$$

$$\rho = 10^{(-RL/20)} \quad (18)$$

4. Approximate (13b) by:

$$\Theta(z = \Delta z) = \beta_o(z)\Delta z - \pi\bar{u}_o/2 \quad (19)$$

where $\Delta z = \lambda/40$, λ is the design wavelength.

and $\beta_o(z)$ is the propagation constant corresponding to impedance Z_L. It is obtained by using a suitable analysis technique. In (19) it is assumed that $\beta_o(z)$ has negligible variation over Δz.

5. Initialize by equating z to zero. The corresponding value of Θ is $-\pi\bar{u}_o/2$.
6. Using $\pi\bar{u}_o$ and $\Theta(z = \Delta z)$ in (13) obtain $Z(\Delta z)/Z_0$
7. Using synthesis and subsequent analysis technique obtain $\beta_o(\Delta z)$

Repeat the above steps until the physical parameter of the transmission line corresponding to $Z(\pi\bar{u}_o/2)$ is reached.

In step (3) one can also choose the electrical length of the taper directly, which alternatively fixes the passband ripple.

DESIGN STEPS FOR 180° ASYMMETRIC COUPLER

Design of asymmetric coupler shown in Figure 3 is based on choosing one of two parameters: the cutoff electrical length $\pi\bar{u}_o$ and the sidelobe level.

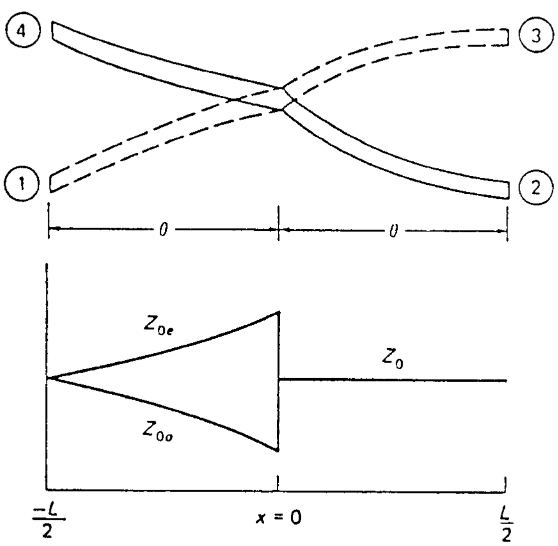

FIGURE 3 : Asymmetric 180° Directional Coupler

1. Given the coupling at infinite frequency C_∞, determine the even mode impedance

$$Z_{oe} = \frac{1 + |C_\infty|}{1 - |C_\infty|}$$

2. From (13) the even mode impedance distribution is obtained. For β_0 (z) take the average of β_{oo} (z) and β_{oe} (z) at each step, where:

   β_{oo} (z) is the odd mode phase constant.

   β_{oe} (z) is the even mode phase constant.

In the above procedure the physical dimensions of the coupled lines are determined using a suitable synthesis technique. β_{oo} (z) and β_{oe} (z) are obtained using an analysis technique.

Currently optimum finline and suspended microstripline tapers and couplers are being fabricated and tested. The design detail and experimental results will be reported in a later submission. Figures 4 and 5 show the computed taper profiles of Ka-band unilateral fin-lines, on 5 mils RT Duroid Substrate, using the above method.

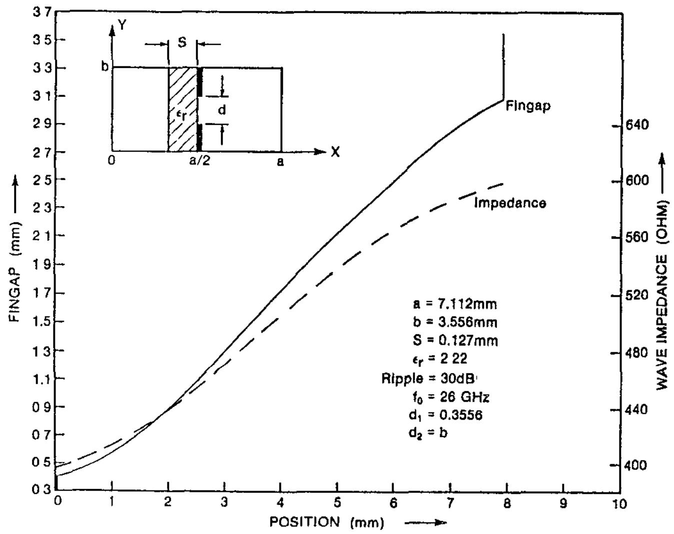

FIGURE 4 : Profile of Finline Taper at Ka-Band

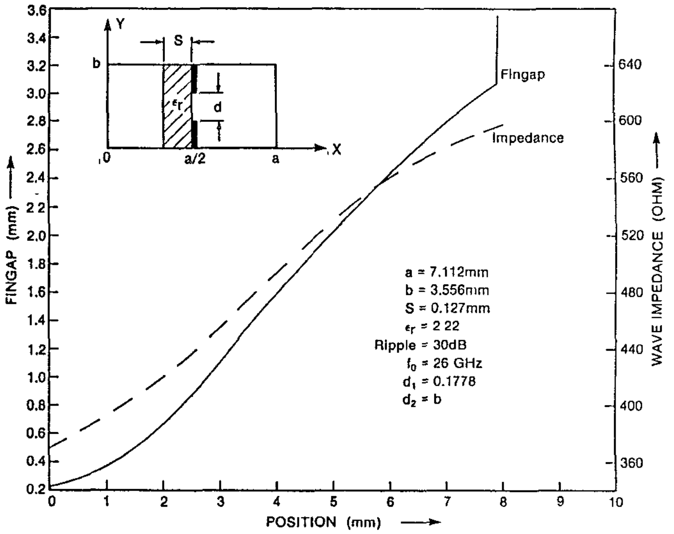

FIGURE 5 : Profile of Finline Taper at Ka-Band

ACKNOWLEDGEMENT

The authors are thankful to the management of COM DEV Ltd. for permission to present this paper.

REFERENCES

1. Bolinder, F., "Fourier Transforms in the Theory of Inhomogeneous Transmission Lines", Proc. IRE, Vol. 38, p. 1354, Nov. 1950.
2. Klopfenstein, R. W., "A Transmission Line Taper of Improved Design", Proc. IRE Vol. 44, pp. 31-35, January 1956.
3. Collin, R. E., "The Optimum Tapered Transmission Line Matching Section", Proc. IRE Vol. 44, pp. 539 - 548, April 1956.
4. Pramanick, P. and Bhartia, P., "Tapered Microstrip Transmission Line", IEEE MTT-S Symposium Digest, pp. 242-244, June 1983.
5. Pramanick, P. and Bhartia, P., "Analysis and Synthesis of Tapered Finlines", IEEE MTT-S Symposium Digest, pp. 336-338, June 1984.
6. Schieblich, C. et al., "Synthesis of Optimum Finline Tapers Using Dispersion Formulas for Arbitrary Slot Widths and Locations", IEEE Trans. MTT 32, No. 12 pp. 1638 - 1645, Dec. 1984.
7. Collin, R. E., "Foundations for Microwave Engineering", McGraw Hill Publications, 1966.